Primary Firm Driven Portfolio Loss

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February 17, 2017

Abstract

Many financial institutions provide loans to secondary firms, whose economic survival depends on the economic condition of primary firms. Even if loans from primary firms are not held in the loan portfolio, financial distress by primary firms can adversely affect the loan portfolio of a financial institution. This paper describes a simple model that can be used for risk management. It directly incorporates the dependence of the conditional probability of default and loss given default of secondary firms on primary firms. Two simple examples show that failure to account for such dependence can result in the value-at-risk and the expected shortfall being greatly under estimated.

Keywords: Primary and secondary firms, Gaussian latent factor model, expected loss, value-at-risk, expected shortfall.

1 Introduction

There are many industries where there are a number of primary firms that employ the services of secondary firms. For many of these secondary firms, their main source of revenue is provided by one or more of these primary firms. Consequently, if a primary firms defaults, the economic consequences for secondary firms that have economic relations can be severe; in extreme cases, it can cause default. In 2002 when Kmart defaulted, Fleming Companies, Inc., which supplied Kmart with all of its groceries, was forced into liquidation - see Turner (2006). After General Motors and Chrysler went bankrupt in 2009 many small suppliers went into liquidation and many employees lost their jobs - see Beene (2009) and Goolsbee and Krueger (2015). Jorion and Zhang (2009) document empirical evidence of the detrimental effects caused by the default of a primary firm. For banks providing loans to firms within such industries, failure to account for the economic structure within the industry may result in under estimation of the credit risk in the loan portfolio, as the presence of secondary firms in a loan portfolio will affect risk metrics such as value-at-risk and expected shortfall.

Many employees of secondary firms have personal loans, such as home and auto loans, with their local banks. If a primary firm suffers economic distress, employees of secondary firms may be laid-off or face reductions in the number of hours they are allowed to work, adversely affecting their credit worthiness. This may affect the credit risk of financial institutions providing the loans to the employees. Financial institutions catering to primary and secondary firms and their employees need to recognize the risk arising from the dependence of secondary firms on primary firms. Many small financial institutions and online firms offering both corporate loans and personal loans have limited risk management facilities and consequently there is the need for a simple model to incorporate this form of risk.

The focus of this paper is to examine how the presence of a primary firm affects the loss distribution of a portfolio of loans to secondary firms. The credit worthiness of a primary firm will affect the probability of default for secondary firms. Furthermore, if a primary firm defaults, this will, in general, affect the distribution of the loss given default of secondary firms. These two effects will adversely affect the value-at-risk and the expected shortfall. We demonstrate that both effects can be substantial. Many credit card holders may have either direct or indirect exposure to a primary firm. Given the number of exposed card holders can be large, this suggests we consider a large homogeneous portfolio case. We derive the conditional asymptotic distribution, drawing on the work of Gordy (2003).

Jarrow and Yu (2001) consider the consequences for counterparty risk in the presence of a primary firm. They extend the reduced form model introduced by Jarrow and Turnbull (1992, 1995), to consider obligors that have correlated defaults because of dependence on economic factors and economic relations. For primary firms, default is driven by economic factors. There are also secondary firms, which have economic relations with primary firms. If a primary firm defaults, this will have an adverse effect on the credit worthiness of secondary firms. However, if a secondary firm defaults, it will have no impact on the credit worthiness of a primary firm. This type of model has been applied by Leung and Kwok (2005) in pricing credit default swaps subject to counterparty risk.

A similar type of problem arises in the pricing of insured debt. Heitfield and Barger (2003), in a discrete time framework, examined the issue of insured debt payments and the implications for regulatory capital. We extend their model to incorporate the impact of failure by a primary firm both on the probability of default and the loss given default. The probability of default and the loss given default vary with the state of the economy - see Frye (2000), Pykhtin (2003), Altman, Brady, Resti and Sironi (2005), Acharya, Bharath and Srinivasan (2007) and Chava, Stefanescu and Turnbull (2011). We allow both the probability of default and the loss given default to vary with the state of the economy and the credit worthiness of the primary firm, implying that defaults and the loss given default will be correlated across firms.

Section two of the paper describes the model for the portfolio loss distribution. Section three considers a large homogeneous portfolio analysis and develops a relatively easy to use result for the loss distribution. Section four examines the practical implications and demonstrates that failure to account for the presence of a primary firm can result in substantial under estimation of the value-at-risk and expected shortfall. The paper ends with a short summary.

2 Model Development

We start by considering two obligors: a primary obligor, denoted by the symbol A and a secondary firm denoted by the symbol S, using the terminology introduced by Jarrow and Yu (2001). The probability of default and the loss given default are both affected by the economy. The starting point for our model is similar to the model described in Heitfield and Barger (2003), who consider guaranteed debt. We extend their model to incorporate stochastic loss given default. If a primary obligor A defaults, then this can have an adverse effect on a secondary obligor S; in the extreme case, S defaults. However, the secondary obligor, S, can default without affecting the primary obligor A. There are four possible states describing the joint default status of A and S. We adapt the Gaussian latent factor model to describe the probability of default over a fixed horizon in the different states and the loss given default, similar in spirit to Pykhtin (2003). Over a fixed time horizon, let D_A denote the event of default for the primary firm and D_A^N the event of no default, with a similar notation employed for the secondary firm.

For the primary firm the probability of default is described by a Gaussian latent factor model described by

$$X_A = \beta_A Z + \sqrt{1-\beta_A^2} e_A \tag{1}$$

where term Z is common to all obligors, e_A is purely idiosyncratic and Z and e_A are independent, identically distributed, zero mean, unit variance, normally distributed random variables. The term β_A is constant and $|\beta_A| \leq 1$. The probability of default is given by

$$P[D_A] = P[X_A \leq C_A] = \Phi(C_A) \tag{2}$$

where $\Phi(.)$ is the cumulative normal distribution function, C_A threshold, assumed known.

For secondary firms the probability of default will depend on the state of the economy and the credit worthiness of the primary firm, as reflected by X_A. Since X_A is a linear combination of the common factor Z and an idiosyncratic component e_A, we write

$$X_S = \beta_S Z + \gamma_S e_A + \sqrt{1-\beta_s^2-\gamma_s^2} e_S \tag{3}$$

where e_S is an independent, purely idiosyncratic normally distribution random variable, with a zero mean unit variance. The coefficient γ_S is non negative and $\beta_S^2 + \gamma_S^2 \leq 1$. Note that the joint distribution for X_A and X_S is bivariate normal, with correlation coefficient

$$\rho = \beta_A \beta_S + \gamma_S \sqrt{1 - \beta_A^2} \tag{4}$$

The probability of the secondary firm defaulting conditional on no default by the primary firm is

$$P[D_S|D_A^N] = P[X_S \leq C_S|D_A^N] = \Phi_2(C_S, -C_A; -\rho)/[1 - \Phi(C_A)] \tag{5}$$

where $\Phi_2(.,.;.)$ is the cumulative bivariate normal distribution function.

If the primary firm has defaulted, we assume that it continues in existence as it tries to reorganize. The default will adversely affect the credit worthiness of the secondary firm; it will affect the default threshold and the structural specification of the latent variable X_S. The exact specification of the new threshold for the secondary firm is far from trivial. Whether default of the primary firm is caused by firm specific factors or economy wide malaise will have differing impact on secondary firms. Some secondary firms will be more diversified and less dependent on the primary firm than other secondary firms. The loan officer must decide in each case how the different secondary firms will be affected. Here we assume that default by the primary firm, for what ever reason, will increase the default threshold - see Ebert and Lutkebohmert (2012). The new threshold is denoted by $\bar{C}_S$. In the limit, if $\bar{C}_S \to \infty$, default by the secondary firm becomes certain.

The relationship of the latent variable to the common factor Z will in general change and expression (3) is assumed to be

$$X_S = \bar{\beta}_S Z + \bar{\gamma}_S e_A + \sqrt{1 - \bar{\beta}_s^2 - \bar{\gamma}_s^2} \bar{e}_s \tag{6}$$

Note that the distribution for X_A and X_S is bivariate normal, with correlation coefficient

$$\bar{\rho} = \beta_A \bar{\beta}_S + \bar{\gamma}_S \sqrt{1 - \beta_A^2} \tag{7}$$

The issue facing the loan office is specifying the new values for the coefficients. In summary, expressions (3) and (6) can be written

$$X_S = \{ \begin{array}{lll} \beta_S Z + \gamma_S e_A + \sqrt{1 - \beta_s^2 - \gamma_s^2} e_S & \text{if} & X_A > C_A \\ \bar{\beta}_S Z + \bar{\gamma}_S e_A + \sqrt{1 - \bar{\beta}_s^2 - \bar{\gamma}_s^2} \bar{e}_s & \text{if} & X_A \leq C_A \end{array} \tag{8}$$

The probability of default by the secondary firm, conditional on default by the primary firm A is given by

$$P[D_S|D_A] = P[X_S \leq \bar{C}_S|D_A] = \Phi_2(\bar{C}_S, C_A; \bar{\rho})/\Phi(C_A)$$

The unconditional probability of default for the secondary firm is given by

$$\begin{aligned} P[D_S] &= P[D_S|D_A^N]P[D_A^N] + P[D_S|D_A]P[D_A] \\ &= \Phi_2(C_S, -C_A; -\rho) + \Phi_2(\bar{C}_S, C_A; \bar{\rho}) \end{aligned} \tag{9}$$

The first term on the right side of the above expression is the probability of a secondary firm defaulting and no default by the primary firm; the second term is the probability of default by the secondary firm and default by the primary firm.

The loss given default (LGD) for the secondary firm is in general effected by the default of the primary firm. If the primary firm has not defaulted, then the LGD per unit of notional is denoted by L_S and if the primary firm has defaulted by $\bar{L}_S$. This is summarized by

$$L = \left\{ \begin{array}{ll} L_S; & \text{no default by primary firm} \\ \bar{L}_S; & \text{default by primary firm} \end{array} \right. \tag{10}$$

If we assume that the loss given default is a constant varying only with the default status of the primary firm, then the expected loss due to default by a secondary firm is given by

$$\begin{aligned} E[L] &= L_S P[D_S,\ D_A^N] + \bar{L}_S P[D_S,\ D_A] \\ &= L_S \Phi_2(C_S, -C_A; -\rho) + \bar{L}_S \Phi_2(\bar{C}_S, C_A; \bar{\rho}) \end{aligned} \tag{11}$$

using expression (9). The first term on the right side is the expected loss arising from default by a secondary firm and no default by the primary firm; the second term is the expected loss arising from default by a secondary firm and the primary firm.

We relax the assumption that the L_S and $\bar{L}_S$ are deterministic. The LGD is defined between zero and one. While a beta distribution is often assumed - see Gupton and Stein (2002), it is well known that the LGD tends to increase, as the probability of default increases. Paragraph 468 of the Basel Framework (2004) requires that the LGD reflects the economic conditions at the time of default (see Basel (2005) for guidance on paragraph 468). Many extant studies assume that recovery rates are dependent on economic and firm specific covariates. In Varma and Cantor (2007) and Acharya, Bharath and Srinivasan (2007) the losses are assumed to be unbounded. Pykhtin (2003) truncates the loss at unity. Schönbucher (2003) and Düllmann and Trapp (2004) assume the loss is described by a logit function, while in Andersen and Sidenius (2004/2005) (AS) a probit function is assumed. Chava, Stenfanescu and Turnbull (2011) find that the logit and probit models are quite similar in out of sample performance. Here we follow AS, the loss given default is described by a probit function:

$$LGD = s[1 - \Phi(\mu + bZ + \sigma\xi)] \tag{12}$$

where ξ is a zero mean, unit variance normally distributed random variable, independent of the common term Z,the term s (≤ 1) represents the maximum loss given default and μ and b are constants. The LGD is bounded $0 \leq L \leq s$. The loan officer must specify the value of μ, the dependence, b, on the common factor, the idiosyncratic volatility σ and the maximum loss given default s. If data are available, the parameters can be estimated using the techniques described in Acharya, Bharath and Srinivasan (2007) or Chava, Stenfanescu and Turnbull (2011). Similar to the beta distribution, the shape of the density function can vary greatly, depending on the values of μ and $\sigma_L^2 = b^2 + \sigma^2$, as shown in AS (2004/2005). The expected value of (12) is given by

$$E[s[1 - \Phi(\mu + bZ + \sigma\xi)]\ ] = s[1 - \Phi(\frac{\mu}{\sqrt{1 + \sigma_L^2}})] \tag{13}$$

as shown in AS (2004/2005).

If there is no default by the primary firm, we assume the loss given default for the secondary firm is given by

$$L_S = s[1 - \Phi(\mu + bZ + \sigma\xi)] \tag{14}$$

The LGD and the probability of default are linked by the common factor Z. Let $Y \equiv \mu + bZ + \sigma\xi$, which is normally distributed with mean μ and variance $\sigma_L^2 = b^2 + \sigma^2$. The latent factor X_S and Y are bivariate normal with correlation coefficient $\rho = \beta_S b/\sigma_L$. If the primary firm has defaulted, then the loss given default for the secondary firm is

$$\bar{L}_S = \bar{s}[1 - \Phi(\bar{\mu} + \bar{b}Z + \bar{\sigma}\bar{\xi})] \tag{15}$$

Let $\bar{Y} \equiv \bar{\mu} + \bar{b}Z + \bar{\sigma}\bar{\xi}$, which is normally distributed with mean $\bar{\mu}$ and variance $\bar{\sigma}_L^2 = \bar{b}^2 + \bar{\sigma}^2$. The latent factors X_S and $\bar{Y}$ are bivariate normal with correlation coefficient $\bar{\rho} = \bar{\beta}_S\bar{b}/\bar{\sigma}_L$.

The loss given default is only realized if the secondary firm defaults. To calculate expected loss given default, we must consider how the primary firm will affect both the probability of default of the secondary firm and the loss given default. The expected loss given default is given by

$$\begin{aligned} E[L|D_S] = E[s(1 - \Phi(\mu + bZ + \sigma\xi))\ |1_{X_S\ \leq\ C_S} 1_{X_A\ >\ C_A}] \\ +E[\bar{s}(1 - \Phi(\bar{\mu} + \bar{b}Z + \bar{\sigma}\bar{\xi}))\ |1_{X_S\ \leq\ \bar{C}_S} 1_{X_A\ \leq\ C_A}] \end{aligned} \tag{16}$$

The first term on the right side is the expected loss conditional on default by the secondary firm and no default by the primary firm and the second term is the expected loss conditional on default by the secondary firm and default by the primary firm. The expected loss is given by

$$\begin{aligned} E[L] &= E[s\Phi(-\mu - bZ - \sigma\xi)\ 1_{X_S\ \leq\ C_S} 1_{X_A\ >\ C_A}] \\ &\quad +E[\bar{s}\Phi(-\bar{\mu} - \bar{b}Z - \bar{\sigma}\xi)\ 1_{X_S\ \leq\ \bar{C}_S} 1_{X_A\ \leq\ C_A}] \end{aligned} \tag{17}$$

which can be written in the form

$$\begin{aligned} E[L] &= s\int_{-\infty}^{\infty} \phi(z)\Phi(\frac{-\mu - bz}{\sqrt{1+\sigma^2}})\Phi_2(\frac{C_S - \beta_S z}{\sqrt{1 - \beta_S^2}},\ \frac{-C_A + \beta_A z}{\sqrt{1 - \beta_A^2}};\ \frac{-\gamma}{\sqrt{1 - \beta_S^2}})\ dz \\ &\quad +\bar{s}\int_{-\infty}^{\infty} \phi(z)\Phi(\frac{-\bar{\mu} - \bar{b}Z}{\sqrt{1+\bar{\sigma}^2}})\Phi_2(\frac{\bar{C}_S - \bar{\beta}_S z}{\sqrt{1 - \bar{\beta}_S^2}},\ \frac{C_A - \beta_A z}{\sqrt{1 - \beta_A^2}};\ \frac{\gamma}{\sqrt{1 - \bar{\beta}_S^2}})\ dz \end{aligned}$$

- see Appendix A for the derivation. This expression can be evaluated using numerical integration. Alternatively the integrals can be written in terms of trivariate normal cumulative distribution functions - see Appendix B for details.

3 Large Homogeneous Portfolios

Consider a portfolio containing loans to n secondary firms over a specified horizon. The notional of each loan is denoted by $N_j,\ j = 1, ..., n$ and the total notional value of the portfolio is $\sum_{j=1}^{n} N_j$. Two simplifying assumptions are made. First, there is only one primary firm, not held in the portfolio, that affects the secondary firms and second, all loans are statistically identical. The portfolio loss per unit of notional is

$$L_P = L_P^{(1)} + L_P^{(2)} \tag{18}$$

where

$$\begin{aligned} L_P^{(1)} &= \tfrac{1}{n}\textstyle\sum_{j=1}^{n} L_j 1_{(X_{j,S} \leq\ C_S)} 1_{(X_A >\ C_A)} \\ L_P^{(2)} &= \tfrac{1}{n}\textstyle\sum_{j=1}^{n} \bar{L}_j 1_{(X_{j,S} \leq\ \bar{C}_S)} 1_{(X_A \leq\ C_A)} \end{aligned}$$

The term $L_P^{(1)}$ is the loss if secondary firms default and the primary firm does not default and L_j is the loss given default of the $j\ th$ secondary firm. The second term $L_P^{(2)}$ is the loss if the primary firm and secondary firms default and $\bar{L}_j$ is the loss given default of the $j\ th$ secondary firm. Note that both the probability of default and the loss given default are impacted by the default of the primary firm. As $L_j,\ j = 1, ..., N$, is defined over the unit interval, its variance is bounded.

Consider the first term on the right side of expression (18). The expected value, conditional on Z and e_A, is given by

$$\begin{aligned} E[L_P^{(1)}|Z, e_A] &= \frac{1}{n}\sum_{j=1}^{n} E(L_j|Z, e_A) E(1_{(X_{j,S}\ \leq\ C_S)}|Z, e_A) 1_{(X_A\ >\ C_A)} \\ &\equiv h(Z, e_A) 1_{(X_A\ >\ C_A)} \end{aligned} \tag{19}$$

using the conditional independence. Drawing on the work of Gordy (2003), we now show that the conditional variance goes to zero, as the size of the portfolio increases. First, given conditional independence, we have

$$var(L_P^{(1)}|Z, e_A) = \frac{1}{n^2}\sum_{j=1}^{n} var(L_j 1_{(X_{j,S}\ \leq\ C_S)}|Z, e_A) 1_{(X_A\ >\ C_A)}$$

recognizing that $1_{(.)}^2 = 1_{(.)}$. Consider

$$\begin{aligned} & var(L_j 1_{(X_{j,S}\ \leq\ C_S)}|Z, e_A] \\ = \ & E[(L_j)^2|Z, e_A] E[(1_{(X_{j,S}\ \leq\ C_S)})^2|Z, e_A] - [E(L_j|Z, e_A)]^2 [E(1_{(X_{j,S}\ \leq\ C_S)}|Z, e_A)]^2 \\ = \ & var(L_j|Z, e_A) E[(1_{(X_{j,S}\ \leq\ C_S)})^2|Z, e_A] + var(1_{(X_{j,S}\ \leq\ C_S)}|Z, e_A)[E[L_j|Z, e_A]]^2 \end{aligned}$$

The second line follows given conditional independence and the third line after simplification. Therefore,

$$\lim_{n\to\infty} var(L_P^{(1)}|Z, e_A) = 0$$

and using Chebychev's inequality, conditional on (Z, e_A), $L_P^{(1)}$ converges in probability to $h(Z, e_A)$.

A similar analysis and assumptions is applied to $L_P^{(2)}$. Again condition on Z and e_A, so that

$$\begin{aligned} E[L_P^{(2)}|Z, e_A] &= \frac{1}{n}\sum_{j=1}^{n} E(\bar{L}_j|Z, e_A)E[1_{(X_{j,S} \,\leq\, \bar{C}_S)}|Z, e_A]1_{(X_A \,\leq\, C_A)} \\ &\equiv \bar{h}(Z, e_A)1_{(X_A \,\leq\, C_A)} \end{aligned}$$

and $L_P^{(2)}$ converges in probability to $\bar{h}(Z, e_A)$.

Given the assumption that all the firms are staistically identical, then

$$E[1_{(X_j \,\leq\, C_S)}|Z, e_A] = \Phi(\frac{C_S - \beta_S Z - \gamma e_A}{\sqrt{1 - \beta_S^2 - \gamma^2}})$$

and

$$E(L_j|Z, e_A) = s[1 - \Phi(\frac{\mu + bZ}{\sqrt{1 + \sigma^2}})]$$

so that

$$h(Z, e_A) = s[1 - \Phi(\frac{\mu + bZ}{\sqrt{1 + \sigma^2}})]\Phi(\frac{C_S - \beta_S Z - \gamma e_A}{\sqrt{1 - \beta_S^2 - \gamma^2}}) \tag{20}$$

and

$$\bar{h}(Z, e_A) = \bar{s}[1 - \Phi(\frac{\bar{\mu} + \bar{b}Z}{\sqrt{1 + \bar{\sigma}^2}})]\Phi(\frac{\bar{C}_S - \bar{\beta}_S Z - \bar{\gamma} e_A}{\sqrt{1 - \bar{\beta}_S^2 - \bar{\gamma}^2}}) \tag{21}$$

The loss distribution is given by

$$\begin{aligned} P[L_P &\leq y] = P[h(Z, e_A)1_{(X_A \,>\, C_A)} + \bar{h}(Z, e_A)1_{(X_A \,\leq\, C_A)} \leq y] \qquad (22) \\ &= P[h(Z, e_A) + (\bar{h}(Z, e_A) - h(Z, e_A))1_{(X_A \,\leq\, C_A)} \leq y] \end{aligned}$$

Expression (22) does not in general lend itself to simplification. We consider a special case. Assume that default by the primary firm does not affect the loss given default and the critical default barrier for the secondary firms ($\bar{C}_S = C_S$), implying $\bar{h}(Z, e_A) = h(Z, e_A)$. This case is still of interest. First, the loss given default and the probability of default for the primary and secondary firms are related through the common dependence on the economic factor Z. Second, the idiosyncratic risk of the primary firm affects the probability of default of secondary firms, assuming the gamma coefficient, γ, is positive. In this case expression (22) simplifies to

$$P[L_P \leq y] = P[h(Z, e_A) \leq y]$$

Let $a(Z) = \{s[1 - \Phi(\frac{\mu+bZ}{\sqrt{1+\sigma^2}})]\}^{-1} > 0$, so that

$$P[L_P \leq y|Z] = P[\Phi(\frac{C_S - \beta_S Z - \gamma e_A}{\sqrt{1 - \beta_S^2 - \gamma^2}}) \leq a(Z)y|Z]$$

Now

$$C_S - \beta_S Z - \sqrt{1 - \beta_S^2 - \gamma^2}\Phi^{-1}(a(Z)y) \leq \gamma e_A$$

Let

$$g(Z; y) \equiv \frac{C_S - \beta_S Z - \sqrt{1 - \beta_S^2 - \gamma^2}\Phi^{-1}(a(Z)y)}{\gamma}$$

so that

$$P[L_P \leq y|Z] = P[g(Z; y) \leq e_A|Z] = \Phi(-g(Z; y))$$

and

$$P[L_P \leq y] = \int_{-\infty}^{\infty} \phi(z)\Phi(-g(z; y))dz \qquad (23)$$

which can be evaluated using numerical integration. If gamma (γ) equals zero, then the above expression simplifies to the result in Proposition 5 in AS (2004/2005).

4 Practical Implications

The risk characteristics of a loan portfolio will depend on the size of each loan, the number of primary firm N_P, the number of secondary firms n_S and the number firms that are not affected by primary firms $\bar{n}_S$. We call these firms non-secondary firms. We consider a loan portfolio with secondary and non secondary firms. There is one primary firm, not held in the loan portfolio, that can affect the secondary firms. The loss due to default can be represented by

$$L_t = \sum_{k=1}^{n_S} N_k L_k 1_{(X_k \leq C_k)} + \sum_{i=1}^{\bar{n}_S} N_i L_i 1_{(X_i \leq C_i)} \qquad (24)$$

The two terms on the right side respectively represent the loss due to default of secondary firms and non-secondary firms in the loan portfolio and N_k and N_i denotes the notional of the respective loans. Note that while the primary firm is not held in the loan portfolio, its presence can still have an adverse effect on the portfolio.

4.1 Example

To provide a benchmark, we start by considering a portfolio of non-secondary firms, assuming the loss given default is a constant. We then extend this analysis by introducing secondary firms. Finally we consider the impact of when the loss given default is stochastic and correlated with the probability of default.

4.1.1 Benchmark

To provide a benchmark, we consider a portfolio of 100 non secondary firms. There are no primary and secondary firms and the LGD is assumed to be known. The probability of default for each firm is assumed to be 2 percent, the face value of debt is 100 for all firms and the loss given default is assumed known at 0.50 per dollar of notional. We start be assuming firms are independent. In expression (1) the beta coefficient is zero ($\beta = 0$), implying no dependence on the common factor Z. The portfolio loss is described by a binomial distribution. Therefore the expected loss is 100 and the standard deviation of the portfolio loss is 70. For the case that ($\beta = 1$), default is driven solely by the common factor Z. There are only two states: no defaults or all firms default and the distribution becomes a Bernoulli distribution. The expected loss is independent of beta - see expression (2) - and remains unchanged at 100. The standard deviation of the portfolio loss increases to 700. This increase in the standard deviation will increase both the value-at-risk and the expected shortfall. The simulation results are shown in Table 1, Case 1. The expect loss remains unchanged, as the beta coefficient increases. However, both the value-at-risk and the expected shortfall increase, as beta increases. This is to be expected, as the nature of the distribution changes.

In Case 2, we assume there are 10 secondary firms and 90 non secondary firms in the portfolio, so the total number of firms remains constant at 100. There is one primary firm, not held in the portfolio, that affects the 10 secondary firms. The probability of default for the primary firm is 1 percent. Initially the probability of default for secondary and non secondary firms is 2 percent. If the primary firm defaults, the probability of the secondary firms increases to 20 percent. We make the simplifying assumption that the event of default by the primary firm does not affect the coefficients in expression (3) - the coefficients in expressions (3) and (6) are identical. What does change is the threshold and the loss given default, which increases from 0.50 to 0.70 per dollar of face value.

When beta of the secondary and non secondary firms equals zero ($\beta = 0$), the expected loss increases to 103.73 compared to 99.95 in Case One, reflecting the impact that the presence of the primary firm has on the conditional probability of default and loss given default for the secondary firms. In the remaining cases, beta for the secondary firms and non-secondary firms is assumed to be identical.

For positive beta, the probability of default for secondary firms and the expected loss are functions of beta - see expressions (9) and (11) respectively. This is not the case for non-secondary firms; the probability of default and the expected loss are independent of beta. However, higher moments in the loss distribution are affected by beta, as Case 1 demonstrates. It is observed that the expected loss increases and then decreases. This is explained by considering the impact on the expected loss, as the beta of the secondary firm increases. The expected loss depends on two terms: the first term is the probability of the secondary firm defaulting and no default by the primary firm and the second term the probability of the secondary and primary firms defaulting – see expression (11). As the beta increases, the correlation between the latent factors X_A and X_S increases - see expression (4). The first probability decreases as the correlation increases, while the second probability increases. Eventually the magnitude of the increases in the second factor is offset by the decreases in the first factor, implying that the expected loss starts to decrease. This effect depends upon the parameter values and the correlation. In Case 4, the expected loss does not decrease. Both the value-at-risk and expected shortfall increase, as beta increases.

Case 3 is similar to Case 2, except there are now 30 secondary firms and 70 non secondary firms. There is one primary firm, not held in the portfolio, that affects the 30 secondary firms. The results are similar to Case 2, though with thirty percent of the portfolio being affected by the primary firm, the effects are more clearly discernible. For the case when beta equals zero, expected loss is now 110.95 compared to 103.73 in Case 2. Similar to Case 2, the expected loss increases as beta increases and then starts to decrease. The value-at-risk and expected shortfall all increase relative to Case 2.

Case 4 is similar to Case 3 except gamma is now zero, implying that the idiosyncratic term of the primary firm does not affect the probability of default of secondary firms – see expression (3). There are two noticeable effects. First, compared to Case 3, the expected loss is lower. For example, when beta equals zero, expected loss is now 103.67 compared to 110.95 in Case 3. Second, as beta increases the expected loss increases, unlike Cases 2 and 3. The magnitude of the correlation is lower compared to Cases 2 and 3 – see expression (4). As the beta increases, the correlation increases. The first probability decreases as the correlation increases, while the second probability increases. However in this case the magnitude of the increases in the second factor is greater than the decreases in the first factor, implying that the expected loss does not decrease. Both the value-at-risk and expected shortfall increase, as beta increases.

4.1.2 Stochastic LGD

The effects of stochastic loss given default are examined in Table 2. To provide a benchmark, similar to Case 1 in Table 1, we consider a portfolio of 100 non secondary firms. The expected loss per unit of total notional, can be written in the form

$$\frac{1}{100}\sum_{j=1}^{100} sE[1_{(X_j \leq C_{NS})}] - sE[\Phi(\mu + bZ + \sigma\xi_j)1_{(X_j \leq C_{NS})}] \qquad (25)$$

The first term on the right side inside the summation is equal to $sP[X_j \leq C_{NS}]$ and is independent of beta. The second term arises because of the stochastic nature of the loss given default. We can write the second term in the form

$$sE[\Phi(\mu + bZ + \sigma\xi_j)1_{(X_j \leq C_{NS})}] = s\Phi_2(\frac{\mu}{\sqrt{1+\sigma^2+b^2}}, C_{NS}; \rho) \qquad (26)$$

where $\rho = -\beta b/\sqrt{1+\sigma^2+b^2}$. The expected loss per unit of total notional can be rewritten in the form

$$\frac{1}{100}\sum_{j=1}^{100} s[\Phi(C_{NS}) - \Phi_2(\frac{\mu}{\sqrt{1+\sigma^2+b^2}}, C_{NS};\rho)] \tag{27}$$

The proof is given in the Appendix C.

To provide a reasonable benchmark, the drift term μ in expression (27) must be defined. The loss given default in Case 1, Table 1 is assumed to 0.50 per unit of face value. We assume that $s = 1$, $b = 0.10$ and $\sigma = 0.35$ and using expression (13), we set μ such that

$$L = s[1 - \Phi(\frac{\mu}{\sqrt{1+b^2+\sigma^2}})] \Rightarrow \frac{\mu}{\sqrt{1+b^2+\sigma^2}} = \Phi^{-1}(1-\frac{L}{s}) \tag{28}$$

where $L = 0.50$. Therefore,

$$E[\Phi(\mu + bZ + \sigma\xi_j)1_{(X_j \leq C_{NS})}] = \Phi_2(\Phi^{-1}(1-\frac{L}{s}), C_{NS};\rho)$$

In Table 1, Case 2, when the primary firm defaults, the loss given default jumped to 0.70 from 0.50. For Case 2, Table 2, we assume $\bar{s} = 1$, $\bar{b} = 0.10$ and $\bar{\sigma} = 0.35$ and we define $\bar{\mu}$ for this case using expression (28), where $L = 0.70$. The probability density functions for the two cases are shown in Figure 1. For the first case when $L = 0.50$, the distribution is symmetric. For the second case, when $L = 0.70$, there is a large probability mass to the right of the mean and a long tail to the left of the mean.

In Case 1, Table 2, there are 100 non-secondary firms. When beta is zero the expected loss is 100.13 and this increases to 113.35 when beta is equal to 0.75. This differs from the results in Case 1, Table 1, where the expected loss was constant. The fact that the expected loss varies with beta is to be expected, given expression (27), where the correlation is proportional to beta. The value-at-risk and expected shortfall have increased, these changes being due to the stochastic of the loss given default.

Cases 2 and 3 are similar in specification to Cases 2 and 3 in Table 1. When beta of the secondary firms is zero, the results are very similar. This is to be expected, as the loss and probability of default are orthogonal. Also, we have normalized the mean of the stochastic loss given default, so that the expected loss is identical to the non-stochastic case. For positive beta, the results however are different, as the probability of default and the loss given default are correlated. First, the expected loss increases as the beta of the secondary firms increases, unlike the results in Table 1. Second, the value-at-risk and expected shortfall values are larger in Table 2. Again this is to be expected given the stochastic nature of the loss given default.

5 Summary

Many financial institutions cater to secondary firms and their employees. Even if loans from primary firms are not held in the loan portfolio, financial distress by primary firms can adversely affect the loan portfolio of a financial institution. This paper describes a simple model that can be used for risk management. It directly incorporates the dependence of secondary firms on primary firms. Two simple examples show that failure to account for such dependence can result in the value-at-risk and the expected shortfall being greatly under estimated.

Declaration of Interest

The author reports no conflicts of interest. The author is solely responsible for the contents and writing of the paper.

Acknowledgements

I thank the referee and the editor for their comments, which have greatly improved the paper.

Table 1

Expected Portfolio Loss

	Beta	0.0	0.25	0.50	0.75
Case 1					
Expected Portfolio Loss		99.95	100.17	99.34	99.76
		(0.10)	(0.09)	(0.14	(0.29)
VaR		287	267	487	$1,066$
Expected Shortfall		325	316	643	$1,417$
Case 2					
Expected Portfolio Loss		103.73	104.31	104.34	103.58
		(0.12)	(0.11)	(0.11)	(0.31)
VaR		351	402	570	$1,091$
Expected Shortfall		468	507	732	$1,467$
Case 3					
Expected Portfolio Loss		110.93	112.67	113.27	112.38
		(0.19)	(0.21)	(0.25)	(0.37
VaR		818	949	$1,105$	$1,367$
Expected Shortfall		$1,082$	$1,156$	$1,336$	$1,743$
Case 4					
Expected Portfolio Loss		104.12	105.67	108.05	110.33
		(0.10)	(0.11)	(0.18)	(0.34)
VaR		319	410	655	$1,238$
Expected Shortfall		411	549	905	$1,621$

The beta coefficient for the primary firm is fixed at 0.50. The number in parenthesis is the standard error of the estimate of the expected loss. The confidence interval is set at 99%. An antithetic Monte Carlo simulation is used.

Case 1. There are no primary firms, 100 non secondary firms. For these non secondary firms, the probability of default is 0.02 and the loss given default 0.50. The face value of debt is 100 for all firms.

Case 2. For Cases 2 and 3, there is one primary firm, not held in the portfolio, that affects the secondary firms. For the primary firm the probability of default is 0.01 and its beta 0.50. There 10 secondary firms that are affected by the primary firm and 90 non secondary firms that are not affected by the primary firm. For the non secondary firms, the probability of default is 0.02 and the loss given default 0.50. For the secondary firms, the probability of default given the primary has not defaulted is 0.02 and the loss given default 0.50. If the primary firms defaults, the probability of default for the secondary firms becomes 0.20 and the loss given default 0.70. For Cases 2 and 3, gamma equals 0.50.

Case 3. There are 30 secondary firms that are affected by the primary firm and 70 non secondary firms that are not affected by the primary firm. All other parameter values remain unchanged.

Case 4. similar to Case 3, except gamma equals zero.

Table 2
Stochastic Loss Given Default
Value-at-Risk and Expected Shortfall

	Beta	0.00	0.25	0.50	0.75
Case 1					
Exp Portfolio Loss		100.13	104.46	109.03	113.35
		(0.10)	(0.10)	(0.17)	(0.35)
VaR		299	310	596	1,295
Exp Shortfall		350	367	799	1,763
Case 2					
Exp Portfolio Loss		103.97	109.02	113.56	117.97
		(0.11)	(0.12)	(0.19)	(0.36)
VaR		368	434	669	1,308
Exp Shortfall		480	548	875	1,774
Case 3					
Exp Portfolio Loss		111.41	117.42	123.00	125.37
		(0.20)	(0.21)	(0.28)	(0.42)
VaR		847	1,007	1,184	1,572
Exp Shortfall		1,105	1,216	1,461	2,013

The beta coefficient for the primary firm is fixed at 0.50. The number in parenthesis is the standard error of the estimate of the expected loss. The confidence interval is set at 99%. An antithetic Monte Carlo simulation is used.

The drift term μ in expression (14), is set such the expected value of expression (14) remains unchanged at 0.50 per unit of face value. We assume that $b = 0.10$ and $\sigma = 0.35$. The drift term $\bar{\mu}$ in expression (15) is set such the the expected value of expression (15) remains unchanged at 0.70 per unit of face value. We assume that $\bar{b} = 0.10$ and $\bar{\sigma} = 0.35$.

Case 1. There are no primary firms, 100 non secondary firms. For these non secondary firms, the probability of default is 0.02. The face value of debt is 100 for all firms.

Case 2. For Cases 2 and 3, there is one primary firm, not held in the portfolio, that affects the secondary firms. There 10 secondary firms that are affected by the primary firm and 90 non secondary firms that are not affected by the primary firm. For the primary firm the probability of default is 0.01 and its beta 0.50. For the non secondary firms, the probability of default is 0.02 and the loss given default 0.50. For the secondary firms, the probability of default given the primary has not defaulted is 0.02. If the primary firms defaults, the probability of default for the secondary firms becomes 0.20. For Cases 2 and 3, gamma equals 0.50.

Case 3. There are 30 secondary firms that are affected by the primary firm and 70 non secondary firms that are not affected by the primary firm. All other parameter values remain unchanged.

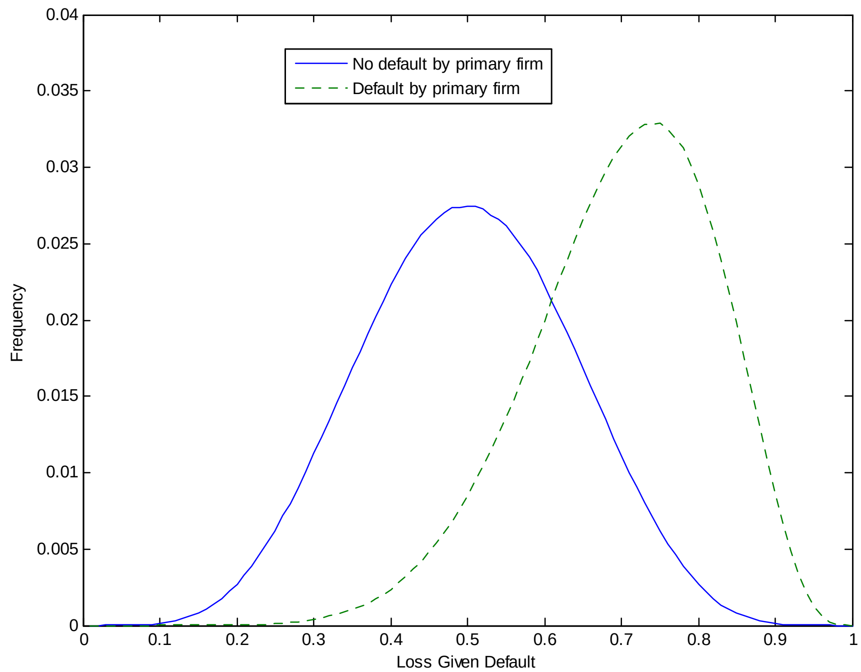

Figure 1
Probability Density Functions for the
Loss Given Default

Appendix A Expected Loss

Consider first the case when the primary firm has not defaulted. We need to consider

$$\Lambda_1 \equiv E[\Phi(-\mu - bZ - \sigma\xi) 1_{X_S \le C_S} 1_{X_A > C_A}]$$

By conditioning on Z and e_A, we can write

$$\begin{aligned}\Lambda_1 &\equiv E\{\ E[\Phi(-\mu - bZ + \sigma\xi) 1_{X_S \le C_S} |\ Z, e_A]\ 1_{X_A > C_A}\} \\ &= E\{\ E[\Phi(-\mu - bZ + \sigma\xi)|\ Z, e_A]\ E[1_{X_S \le C_S} |\ Z, e_A]\ 1_{X_A > C_A}\}\end{aligned}$$

The second line follows given conditional independence. Using the result in Andersen and Sidenius (P. 65), we have

$$E[\Phi(-\mu - bZ + \sigma\xi)|\ Z, e_A] = \Phi(\frac{-\mu - bZ}{\sqrt{1+\sigma_1^2}})$$

Next, conditional on no default by the primary film

$$E[1_{X_S \le C_S} |\ Z, e_A] = \Phi(\frac{C_S - \beta_S Z - \gamma e_A}{\sqrt{1 - \beta_S^2 - \gamma^2}})$$

so that

$$\begin{aligned}\Lambda_1 &= E[\ \Phi(\frac{-\mu - bZ}{\sqrt{1+\sigma^2}})\ \Phi(\frac{C_S - \beta_S Z - \gamma e_A}{\sqrt{1 - \beta_S^2 - \gamma^2}})\ 1_{X_A > C_A}] \\ &= \int_{-\infty}^{\infty} \phi(z) \Phi(\frac{-\mu - bZ}{\sqrt{1+\sigma^2}}) \int_{\frac{C_A - \beta_A z}{\sqrt{1-\beta_A^2}}}^{\infty} \Phi(\frac{C_S - \beta_S z - \gamma e_A}{\sqrt{1 - \beta_S^2 - \gamma^2}}) \phi(e_A) de_A dz \\ &= \int_{-\infty}^{\infty} \phi(z) \Phi(\frac{-\mu - bZ}{\sqrt{1+\sigma^2}}) \Phi_2(\frac{C_S - \beta_S z}{\sqrt{1-\beta_S^2}},\ \frac{-C_A + \beta_A z}{\sqrt{1-\beta_A^2}};\ \frac{-\gamma}{\sqrt{1-\beta_S^2}})\ dz\end{aligned}$$

where $\phi(.)$ is the probability density function of a zero mean, unit variance normally distributed random variable.

For the case when the primary firm has defaulted, we need to consider

$$\Lambda_2 \equiv E[\Phi(-\bar{\mu} - \bar{b}Z + \bar{\sigma}\xi) 1_{X_S \le \bar{C}_S} 1_{X_A \le C_A}]$$

Hence

$$\Lambda_2 = \int_{-\infty}^{\infty} \phi(z) \Phi(\frac{-\bar{\mu} - \bar{b}z}{\sqrt{1+\bar{\sigma}^2}}) \Phi_2(\frac{\bar{C}_S - \bar{\beta}_S z}{\sqrt{1-\bar{\beta}_S^2}},\ \frac{C_A - \beta_A z}{\sqrt{1-\beta_A^2}};\ \frac{\gamma}{\sqrt{1-\bar{\beta}_S^2}})\ dz$$

The expected loss is

$$\begin{aligned}&s \int_{-\infty}^{\infty} \phi(z) \Phi(\frac{-\mu - bz}{\sqrt{1+\sigma^2}}) \Phi_2(\frac{C_S - \beta_S z}{\sqrt{1-\beta_S^2}},\ \frac{-C_A + \beta_A z}{\sqrt{1-\beta_A^2}};\ \frac{-\gamma}{\sqrt{1-\beta_S^2}})\ dz \\ &+\bar{s} \int_{-\infty}^{\infty} \phi(z) \Phi(\frac{-\bar{\mu} - \bar{b}Z}{\sqrt{1+\bar{\sigma}^2}}) \Phi_2(\frac{\bar{C}_S - \bar{\beta}_S z}{\sqrt{1-\bar{\beta}_S^2}},\ \frac{C_A - \beta_A z}{\sqrt{1-\beta_A^2}};\ \frac{\gamma}{\sqrt{1-\bar{\beta}_S^2}})\ dz\end{aligned}$$

Appendix B

Lemma

$$\begin{aligned}&\int_{-\infty}^{\infty}\phi(z)\Phi(a_1+c_1z)\Phi_2(a_2+c_2z,a_3+c_3z;\rho)dz\\ =\ &\Phi_3(\frac{a_1}{\sqrt{1+c_1^2}},\frac{a_2}{\sqrt{1+c_2^2}},\frac{a_3}{\sqrt{1+c_3^2}};\Omega)\end{aligned}$$

The elements of the correlation matrix, Ω, are defined below.

Proof

Let

$$\begin{aligned}x_1 &= -c_1z+\delta_1\\ x_2 &= -c_2z+\delta_2\\ x_3 &= -c_3z+(d_2\delta_2+\delta_3)/\sqrt{1+d_2^2}\end{aligned}$$

where δ_j are independent, identically normally distributed zero mean, unit variance random variables and independent of z. Let $\delta_4=(\delta_3+d_2\delta_2)/\sqrt{1+d_2^2}\sim N(0,1)$, and $\hat{x}_j\equiv x_j/\sqrt{1+c_j^2}\sim N(0,1)$, $j=1,2,3$. The correlation between δ_2 and δ_4 denoted by $\rho(\delta_2,\delta_4)=d_2/\sqrt{1+d_2^2}=\rho$. The correlation elements of Ω are given by $\rho(\hat{x}_1,\hat{x}_2)=c_1c_2/[\sqrt{1+c_1^2}\sqrt{1+c_2^2}]$, $\rho(\hat{x}_1,\hat{x}_3)=c_1c_3/[\sqrt{1+c_1^2}\sqrt{1+c_3^2}]$ and $\rho(\hat{x}_2,\hat{x}_3)=[c_2c_3+\frac{d_2}{\sqrt{1+d_2^2}}]/[\sqrt{1+c_2^2}\sqrt{1+c_3^2}]$.

Consider

$$\begin{aligned}P(x_1 &\le a_1,x_2\le a_2,x_3\le a_3)\\ &= \Phi_3(\frac{a_1}{\sqrt{1+c_1^2}},\frac{a_2}{\sqrt{1+c_2^2}},\frac{a_3}{\sqrt{1+c_3^2}};\Omega)\end{aligned}\tag{29}$$

Now

$$\begin{aligned}P(x_1 &\le a_1,x_2\le a_2,x_3\le a_3)\\ &= E[P(x_1\le a_1|z)P(x_2\le a_2,x_3\le a_3|z)]\\ &= E[P(\delta_1\le a_1+c_1z|z)P(\delta_2\le a_2+c_2z,\delta_4\le a_3+c_3z|z)]\\ &= \int_{-\infty}^{\infty}\phi(z)\Phi(a_1+c_1z)\Phi_2(a_2+c_2z,a_3+c_3z;\rho)dz\end{aligned}\tag{30}$$

The second line follows because of conditional independence. Equating (29) and (30) gives the result.

Appendix C

We want to evaluate

$$
\begin{aligned}
E[\Phi(\mu + bZ + \sigma\xi_j)1_{(X_j \le C_{NS})}] &= E\{E[\Phi(\mu + bZ + \sigma\xi_j)1_{(X_j \le C_{NS})}|Z]\} \\
&= E[\Phi(\frac{\mu + bZ}{\sqrt{1+\sigma^2}})\Phi(\frac{C_{NS} - \beta Z}{\sqrt{1-\beta^2}})]
\end{aligned}
$$

Lemma

$$
\int_{-\infty}^{\infty} \phi(z)\Phi(a_1 + b_1 z)\Phi(a_2 + b_2 z)dz = \Phi_2(\frac{a_1}{\sqrt{1+b_1^2}}, \frac{a_2}{\sqrt{1+b_2^2}}; \rho)
$$

Proof

Let $z_1 = -b_1 z + \delta_1$ and $z_2 = -b_2 z + \delta_2$, where z, δ_1 and δ_2 are zero mean, unit variance independent normally distributed random variables. Now z_1 and z_2 are bivariate normally distributed, with zero means, variances $1 + b_1^2$ and $1 + b_2^2$ and correlation $\rho = b_1 b_2 / [(1 + b_1^2)(1 + b_2^2)]^{1/2}$. Therefore

$$
P(z_1 \le a_1, z_2 \le a_2) = \Phi_2(\frac{a_1}{\sqrt{1+b_1^2}}, \frac{a_2}{\sqrt{1+b_2^2}}; \rho)
$$

Now

$$
\begin{aligned}
P(z_1 &\le a_1, z_2 \le a_2) = E[P(z_1 \le a_1, z_2 \le a_2 | z)] \\
&= E[P(z_1 \le a_1 | z) P(z_2 \le a_2 | z)] \\
&= E[P(\delta_1 \le a_1 + b_1 z \mid z) P(\delta_2 \le a_2 + b_2 z \mid z)] \\
&= \int_{-\infty}^{\infty} \phi(z)\Phi(a_1 + b_1 z)\Phi(a_2 + b_2 z)dz
\end{aligned}
$$

■

By comparison with expression (26), we have

$$
a_1 = \tfrac{\mu}{\sqrt{1+\sigma^2}} \qquad b_1 = \tfrac{b}{\sqrt{1+\sigma^2}}
$$

and

$$
a_2 = \tfrac{C_{NS}}{\sqrt{1-\beta^2}} \qquad b_2 = -\tfrac{\beta}{\sqrt{1-\beta^2}}
$$

implying that

$$
\frac{a_2}{\sqrt{1+b_2^2}} = C_{NS}
$$

and

$$
\rho = -\beta \frac{b_1}{\sqrt{1+b_1^2}}
$$

Finally

$$
E\{s[1 - \Phi(\mu + bZ + \sigma\xi_j)]1_{(X_j \le C_{NS})}\} = s[\Phi(C_{NS}) - \Phi_2(\frac{\mu}{\sqrt{1+\sigma^2+b^2}}, C_{NS}; \rho)]
$$

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